Prioritization of Stockpile Maintenance with Layered Pareto Fronts

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Motivations

• How to make an informed, data-driven decision when decision-makers have competing interests?

• General problem description:
  – Use multiple criteria to identify the top N solutions to accomplish a goal while enabling a richer and more realistic decision
  – Given: Enumerated list of objects evaluated on multiple criteria
  – Goal: Select N objects from the list
  – How do you choose “the best” N while balancing competing objectives?
Stockpile Prioritization Problem

• 42 stockpiles across 4 groups, all could use extra funding
• Primary question of interest: Which stockpiles should receive additional funding?
• The challenges:
  – Limited funds available: only **4** stockpiles can receive $$
  – How should **most critical** be measured? What data needs to be collected?
  – **Competing objectives** of stockpile managers → disagreement on how to prioritize criteria
• Cost of maintenance/surveillance similar across the different stockpiles
• 5-member decision-making team
  – The 4 stockpile group managers
  – Sponsor

<table>
<thead>
<tr>
<th>Group</th>
<th>Number of stockpiles</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>11</td>
</tr>
<tr>
<td>C</td>
<td>11</td>
</tr>
<tr>
<td>D</td>
<td>5</td>
</tr>
</tbody>
</table>
Stockpile Prioritization Problem

• Budget allocation problem among stockpile programs to enhance stockpile performance

• Historic decision-making process
  – Managers of 4 stockpile groups argued with stakeholder
  – Decision often driven by presentation effectiveness and not data

• Goal: Improve the decision process to better allocate the stockpiles in most need
Structured Decision Making Using DMRCS

<table>
<thead>
<tr>
<th>Stage</th>
<th>Steps</th>
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</thead>
<tbody>
<tr>
<td>Objective</td>
<td></td>
</tr>
<tr>
<td>• Define</td>
<td>Identify choices under consideration</td>
</tr>
<tr>
<td></td>
<td>Identify aspects of the decision which are most important</td>
</tr>
<tr>
<td>• Measure</td>
<td>Identify quantitative metrics to characterize aspects</td>
</tr>
<tr>
<td></td>
<td>Gather relevant data for each metric for all choices</td>
</tr>
<tr>
<td>• Reduce</td>
<td>Eliminate some criteria from further consideration</td>
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<tr>
<td></td>
<td>Eliminate non-contending choices</td>
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<tr>
<td>Subjective</td>
<td></td>
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<tr>
<td>• Combine</td>
<td>Evaluate tradeoffs between choices</td>
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<tr>
<td></td>
<td>Incorporate subjective weighting of criteria for all team members</td>
</tr>
<tr>
<td>• Select</td>
<td>Identify top solutions</td>
</tr>
<tr>
<td></td>
<td>Explore performance of top choices relative to competitors</td>
</tr>
<tr>
<td></td>
<td>Finalize choices and how process can be defended</td>
</tr>
</tbody>
</table>

Define

• 42 stockpiles are under consideration

• Brainstorm which criteria should be used:
  – What would lead to a good decision?
  – Brainstorming allows consideration of many different facets of the stockpiles

• Metrics Discussed
  • Various reliability metrics
  • Current supply relative to projected need
  • Availability of suitable alternatives
  • Impact on missions

Excluded:
Too hard to collect objective data or not as essential

• Number of historical failure problems
• SME knowledge on problems with usage
• Quality of reliability testing procedures
Measure

• Determine how each criterion will be defined and measured
• Want quantitative metrics for fair assessments
• SMEs defined what characteristics to define with each of the measures
• Reduced aspects of interest into three broad categories:
  – Overall Reliability
  – Overall Urgency
  – Consequence
Measure

- **Overall Reliability**
  - Current reliability (1.a)
  - Time to threshold (1.b)

- **Overall Urgency**
  - Available supply
  - Availability of alternatives

- **Consequence**
  - How important is munition to mission?
  - Shortage assessment

Each overall criterion given a score from 0 to 10 (10 = most critical)
Measure

• Scoring method based on historical precedent
• Based on comparing available data for each stockpile
• Assessed via standardized definitions, defined by SMEs (independent from decision-making team)
• Each stockpile was assessed by several experts to obtain final score for each metric
• Objectivity in assessing scores is key → eliminates potential bias
• Rigorous and defensible method
  – Incorporates expert knowledge based on all aspects of stockpile design, maintenance, and surveillance
  – Labor-intensive, but transparent and consistent

Decisions are now based on quantitative & objective measures agreed upon by team
### Measure: Subset of Stockpile Criteria Scores

<table>
<thead>
<tr>
<th>Stockpile</th>
<th>Current Reliability</th>
<th>Time to Threshold</th>
<th>Overall Reliability</th>
<th>Available Supply</th>
<th>Availability of Alternate</th>
<th>Overall Urgency</th>
<th>Consequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>8.5</td>
<td>9.5</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>6</td>
<td>2.5</td>
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<td>A2</td>
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<td>8.75</td>
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<td>9.5</td>
<td>8.5</td>
<td>9</td>
<td>8.5</td>
<td>9</td>
<td>8.75</td>
<td>9</td>
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<td>B1</td>
<td>6.5</td>
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<td>5.5</td>
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<tr>
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<td>9.5</td>
<td>9.5</td>
<td>9.5</td>
<td>8</td>
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</tbody>
</table>

**Overall Reliability**
- Min = 5.75
- Max = 9.5

**Overall Urgency**
- Min = 6
- Max = 9.5

**Consequence**
- Min = 2.5
- Max = 9.5

Note: Due to proprietary nature of data, this data is notional (but representative of original data)
Reduce

• Reduce metrics (done previously)
• Reduce non-contending solutions from further consideration to make decision more manageable
• Can we objectively remove some stockpiles with values not critical enough before subjective choices are made?
  – Use layered Pareto Fronts
  – TopN-PFS Add-In for JMP

Pareto Front

• The set of non-inferior points in a dataset
• Goal: Eliminate dominated, inferior points from further consideration
• Balance multiple criteria simultaneously
• The set of non-inferior points in a dataset
• Goal: Eliminate dominated, inferior points from further consideration
• Balance multiple criteria simultaneously
• Note: Ideally would like to be at the Utopia point
Pareto Front

• The set of non-inferior points in a dataset
• Goal: Eliminate dominated, inferior points from further consideration
• Balance multiple criteria simultaneously
• Note: Ideally would like to be at the Utopia point
Layered Pareto Front

- Find points on the Pareto Front
Layered Pareto Front

• Find points on the Pareto Front
• Remove these from obtainable criterion region
Layered Pareto Front

- Find points on the Pareto Front
- Remove these from obtainable criterion region
- Find a new Pareto Front based on the remaining points
Layered Pareto Front

- Find points on the Pareto Front
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Layered Pareto Front

- Find points on the Pareto Front
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- Remove these from obtainable criterion region
- Continue until have m layered Pareto Fronts
Layered Pareto Front

- Find points on the Pareto Front
- Remove these from obtainable criterion region
- Find a new Pareto Front based on the remaining points
- Remove these from obtainable criterion region
- Continue until have \( m \) layered Pareto Fronts
Layered Pareto Front

- Find points on the Pareto Front
- Remove these from obtainable criterion region
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• Find points on the Pareto Front
• Remove these from obtainable criterion region
• Find a new Pareto Front based on the remaining points
• Remove these from obtainable criterion region
• Continue until have m layered Pareto Fronts
Reduce: Stockpile

- 26/42 stockpiles in 4 PF layers
- Objectively eliminate 16 (approx. 1/3) stockpiles from further consideration
- Why PF layers?
  - Consider stockpile A4: \((\text{OR,OU,C}) = (9,8.75,9)\)
  - A4 not in first PF layer; dominated by A3 \((\text{OR,OU,C}) = (9.5,8.75,9)\)
  - But still fairly critical
• Evaluate tradeoffs among contenders using desirability functions
• Make diverse criteria (potentially measured on different scales) comparable

\[
Add\ DF_j = \sum_{i=1}^{k} w_i d_{ij}
\]

\[
Multi\ DF_j = \prod_{i=1}^{k} d_{ij}^{w_i}
\]

\[
\Sigma w_i = 1; w_i \geq 0
\]

• \(d_{ij}\) are desirability scores; map original values to \([0,1]\) scale
• Weights are subjective & may change top N rankings
  – Must reflect decision-makers’ priorities (which likely differ)

• Several choices team must make:
  – How to scale original criteria values?
  – Which desirability function?
  – What values of weights for each criterion?

• TopN-PFS Add-In provides several options to meet decision-makers’ needs

• Scaling:
  1. Use the natural range of each metric; map 10 (most critical) to \( d_{ij} = 1 \) and 0 (least critical) to 0
  2. Use the range of the observed data; e.g., for Overall Reliability, 5.75 → 0, and 9.5 → 1
  3. Use the range of data on the top N PFs layers; e.g., for Overall Reliability, 6.25 → 0, and 9.5 → 1

Scaling choice does not affect solutions on PFs
Combine

• Several choices team must make:
  – How to scale original criteria values?
  – Which desirability function?
  – What values of weights for each criterion?

• TopN-PFS Add-In provides several options to meet decision-makers’ needs

• Desirability Function:
  – Multiplicative DF more severely penalizes low criterion values than the additive DF
  – Additive DF is more forgiving for poor performance of one criterion

Did not want to eliminate potentially critical stockpile
Evaluated impact of DF choice
Combine

• Several choices team must make:
  – How to scale original criteria values?
  – Which desirability function?
  – What values of weights for each criterion?

• TopN-PFS Add-In provides several options to meet decision-makers’ needs

• Weights
  – First consider which are most critical choices across all weights
  – 4 stockpiles identified as most critical across all weights
  – Both stockpiles with top individual scores and those with balanced high scores are identified as critical
Select – Deeper Dive Across All Weights

Goal: Identify top 4 critical stockpiles

For \((OR, OU, C) = (0.35, 0.25, 0.4)\), most critical stockpiles are: A3, A4, B4, D4

\[(wt \text{ Overall Urgency} = 1 - wt \text{ Overall Reliability} - wt \text{ Consequence})\]

Where \((wt \text{ Consequence} = 0.4)\)
Select – Deeper Dive
Across All Weights

Goal: Identify top 4 critical stockpiles

For \((OR, OU, C) = (0.30, 0.30, 0.4)\), A4 & B4 are tied for 2\(^{nd}\) most critical
Select – Deeper Dive Across All Weights

Goal: Identify top 4 critical stockpiles

For (OR, OU, C) = (0.50, 0.10, 0.4), A3, A4, B7, and B11 are most critical

(wt Overall Urgency = 1 - wt Overall Reliability - wt Consequence)

Where (wt Consequence = 0.4)
Select – Deeper Dive Across All Weights

How often is each stockpile in the top 4 across all weights?

- A3, B4, A4 are clear winners
- Recall: A4 is on the 2\textsuperscript{nd} PF layer
- How to choose 4\textsuperscript{th} stockpile?
- If team members agreed on the exact weighting of each criterion, arrive at easy answer

Only 14 stockpiles ever in the top 4 for scaling & DF selections. Combining criteria into overall score has further reduced choices.

No clear winner for 4\textsuperscript{th} stockpile without additional decisions.
Select – Narrow Weight Region

- Team members narrowed down to universally agreeable region of weights
  - Overall Reliability ≥ 20%
  - Overall Urgency ≥ 10%
  - Consequence ≥ 30%

D4 selected as 4th stockpile
Examining the Final Solution

Synthesized Efficiency Plot – how do solutions compare to best possible solution at given weight combo?

- A3 is most critical, so darkest shade of blue
- A3 has dark blue across all weight region shown: near optimal performance regardless of weight
- A4, B4, and D4 also dark blue -> DF score close to most critical (A3)
Was 4 the right choice?

- There is evidence that >4 stockpiles would greatly benefit from funding
- N+1 comparison plot shows ratio of DF scores for (N+1)th solution vs Nth best solution
- High value means (N+1)th solution is close to Nth best
- For N = 4, there is very little difference between 4th most critical and 5th most critical stockpile across weights
- Evidence for sponsor to find additional funding??

Where (wt Consequence = 0.4)
Final Decision

• A3, B4, A4, and D4 were selected as recipients of the additional funding

• All team members agreed that the right decision had been made (although manager C did not receive any additional funding)

• Subjective choices made can be assessed using the JMP Add-In
  – Can quickly recreate the analysis with a different desirability function and/or scaling approach
  – The final stockpile selections in this example were robust to the choices of DF and scaling
JMP Add-In

When identifying an "optimal" choice among many alternatives, we must often balance multiple, and potentially competing objectives. In addition to this problem, in many situations, we often do not want to find just one optimal solution, but multiple solutions. Two scenarios where this could happen are:
- We have a goal or task that requires several "winners."
- We have both quantitative and qualitative objectives. We can reduce the number of options from which to choose to a more manageable number by quantitatively using the quantitative objectives and then make a final decision using the secondary, qualitative criteria.

If you want to identify more than one optimal solution, say the top N, you can use a two-phase Layered Pareto Front approach [1]. In the first phase, we reduce the number of options from which to choose to a more manageable number using Pareto Front layers. In the second phase, we use desirability functions [2] to rank the choices for different combinations of weights on the criteria.

To use this tool, you must begin with a data table listing the enumerated values for multiple criteria and an ID column.

Example
Suppose you are a stockpile manager and need to identify the 4 most critical subpopulations in the stockpile in order to dedicate resources for their maintenance. You have three criteria that have been evaluated: reliability, urgency, and severity of consequence. To simplify this illustration, we only consider reliability and urgency.
Summary

- Using a structured decision process (DMRCS) allowed for a defensible and rigorous process
- Facilitate decision making for a complex process with many criteria
- Better buy-in on final decision because of participation in Define-Measure steps
- Longer decision process, but more informed & data-driven decision is made
Questions?