QUANTITATIVE MEASURES TO EVALUATE PROCESS STABILITY AND ASSESS PROCESS HEALTH

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OUTLINE

- Motivation
- Overview of Process Stability Metrics
- Applications
  - Stability Metrics to Inform Nelson Violations on Shewhart Charts
  - Combining Process Stability and Process Capability
    - Process Performance Graph in JMP®
- Summary
- References & Acknowledgements

MOTIVATION

- Develop a more objective and consistent way to determine if a parameter is in a state of statistical control.
- Evaluate the process stability of hundreds of a parameters in an efficient manner.
- Prioritize and track improvement efforts using the two important dimensions of process health: stability and capability.

PROCESS STABILITY METRICS

- Three metrics were developed:
  - SR Test: short term vs. long term variation
  - ANOVA F-Test: between vs. within variation
  - Instability Ratio (I_{NSR}): Western Electric rule violations
- These metrics are used to classify a parameter as:
  - Stable → process is operating with common cause variation
  - Unstable → process is operating with special cause variation
- Ramírez, B and Runger, G. (2006).

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SR TEST: SHORT TERM VS. LONG TERM VARIATION

- Using different estimates for the population std. dev. has been studied, e.g., Podolski (1989), Cruthis and Ridgon (1992), Wheeler (1995).
- In the context of SPC:
  - Long term variation can be estimated using the sample standard deviation, s
  - Short term variation can be estimated using control chart estimates of standard deviation, such as $\overline{R} \div d_2$ or $\overline{MR} \div d_2$.
- When evaluating process variation:
  - Stable process ► long term variation ≈ short term
  - Unstable process ► long term variation > short term

STABILITY METRIC: SR TEST

- $SR = s^2_{\text{long term}} \div s^2_{\text{short term}} = s^2 \div (\overline{MR} \div d_2)^2$
- Significance Test:
  - H_0: SR = 1 (Stable)
  - H_1: SR > 1 (Unstable)
  - $SR \sim F_{(N-1),\ 0.62(N-1)}$
- Aligns with F Max Test (Hartley, 1950), which is used to determine if two variances are equal using the ratio of the larger variance to the smaller variance.

Cruthis and Ridgon suggest that F (SR) does not follow an F-distribution, but also point out that “the true distribution of F* is difficult or impossible to determine analytically”.*

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SR TEST XmR EXAMPLE

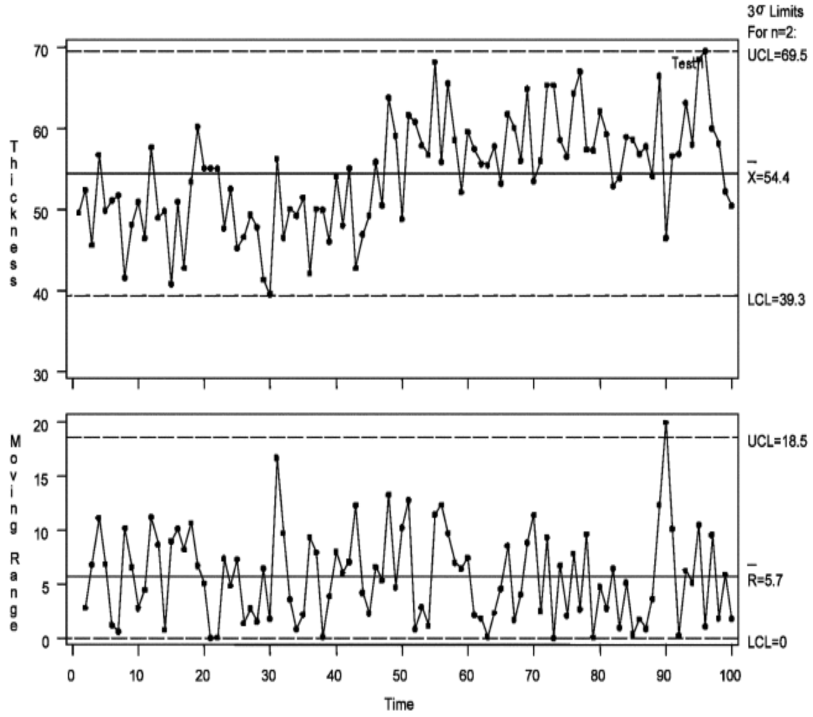

Figure 2. XmR chart of 100 thickness observations example 1.

Ramirez and Runger (2006)

Table 2

Calculations for SR test for example 1

Component	Value
$S^2_{method1} = \sum_i (X_i - \overline{X})^2 / (100 - 1), i = 1 \text{ to } 100$	45.212
$S^2_{method2} = (\overline{MR}/d_2)^2$	$(5.7 \div 1.128)^2 = 25.24$
$SR = S^2_{method1} + S^2_{method2}$	$45.212 \div 25.24 = 1.79$
$df_{method1} = (100 - 1)$	99
$df_{method2} = 0.62 \times (100 - 1)$	62
p-value for 1.79, 99, 62 using *F*-distribution	0.0072

STABILITY METRIC: ANOVA F-TEST

- Compare “within” subgroup variation to “between” subgroup variation to detect differences in subgroup means.
- Natural extension of how control limits for $\overline{X}$R and $\overline{X}$S charts are created.
- Significance Test:
  - H_0: $\mu_1 = \mu_2 = \ldots \mu_k$ (Stable)
  - H_1: at least two means are not equal (Unstable)
  - $F \sim F_{(k\text{-}1),\, k(n\text{-}1)}$
- ANOVA F-test:
  - Well defined and documented
  - We can control the type I error rate

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ANOVA F-TEST XMR EXAMPLE

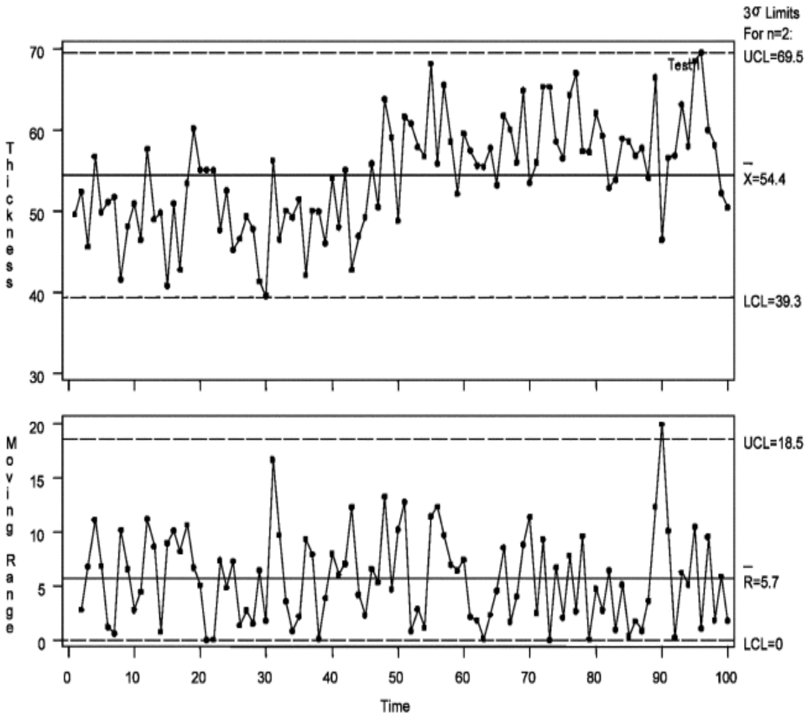

Figure 2. XmR chart of 100 thickness observations example 1.

Ramirez and Runger (2006)

- For XmR charts, an artificial subgroup is created, where consecutive groups of size n are formed.

Table 9

ANOVA for XmR example 1

Source of variation	Sums of squares	Degrees of freedom	Mean squares	Computed F	*P*-value
"Between" subgroups	3181.49	49	64.93	2.51	0.0008
"Within" subgroups	1294.52	50	25.89		
Total	4476.01	99			

STABILITY METRIC: INSTABILITY RATIO

- Patterns of variation:
  - Stable: common cause variation, random about a mean, no unusual patterns or trends
  - Unstable: special cause variation, unusual patterns and trends
- Based upon runs tests violations, such as, Western Electric rules or Nelson rules:
  - 1 point outside of 3σ limits
  - 8 points in a row on one side of the center line
  - 2 of 3 points 2σ and beyond
  - 4 of 5 points 1σ and beyond
- I_{NSR}: Ratio of the number of subgroups with ≥1 violations to the total number of subgroups

INSTABILITY RATIO XmR EXAMPLE

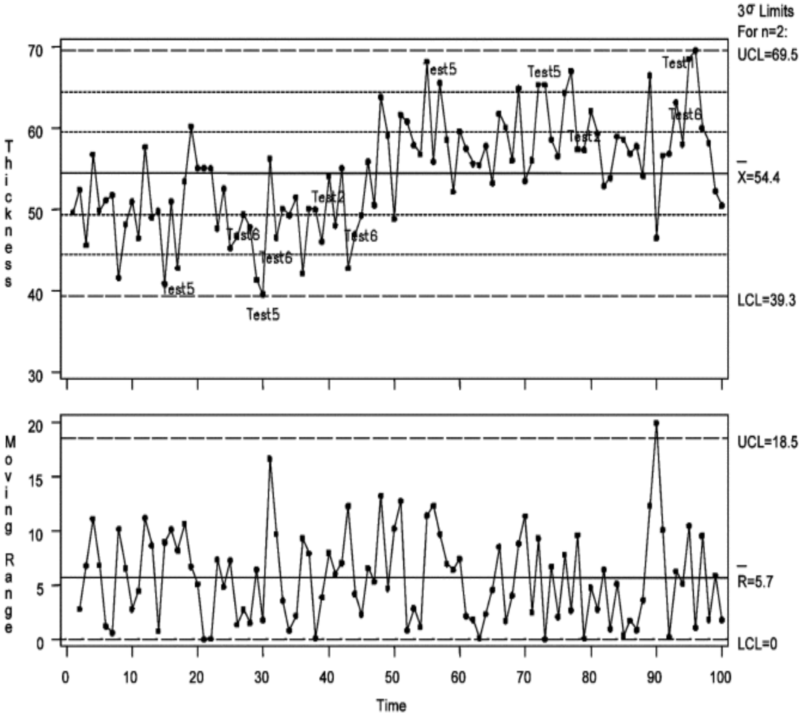

Figure 8. XmR chart with rules turned on example 1.

Ramirez and Runger (2006)

An informal test was proposed: Unstable process if INSR > 4.5%

- 11 subgroups with one ore more violations
- INSR = 100x(11/100) = 11%
- Since 11% > 4.5%, the process is unstable

STABILITY METRIC PERFORMANCE

- Statistical performance has been documented, e.g., Cruthis and Ridgon (1992), Ramirez and Runger (2006), Greene (2007), Britt, Ramirez and Mistretta (2016)

- Conditions studied:
  - Random noise, different sample sizes
  - Special cause variation, different sample sizes and shifts

STATISTICAL PERFORMANCE OF SELECTED METRICS

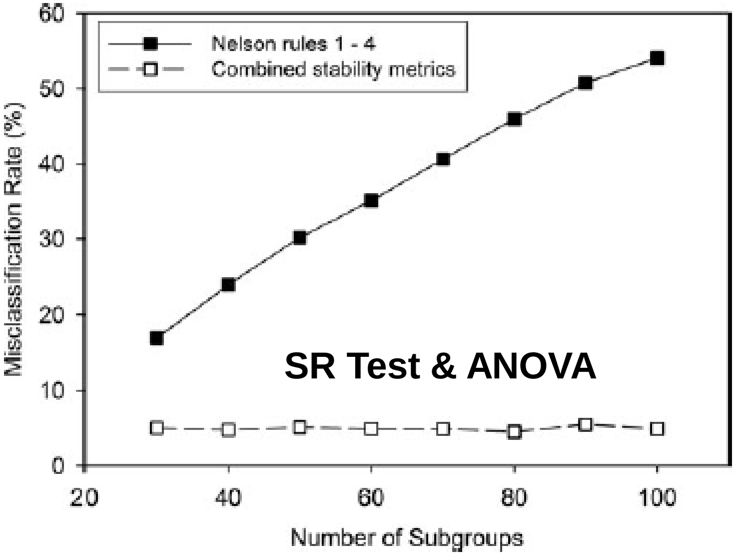

Figure 3. Misclassification rate (i.e., false positives; y-axis) calculated from 10,000 simulations as a function of the number of subgroups (x-axis). The results of simulations using XmR charts with Nelson rules 1–4 and stability metrics are compared.

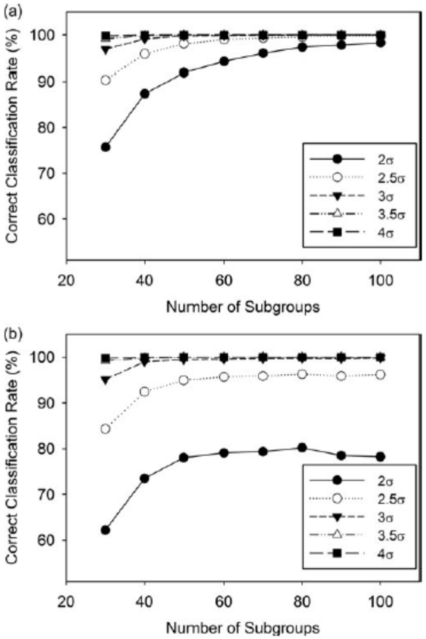

Figure 4. Correct classification rate (i.e., power; y-axis) calculated from 10,000 simulations using (A) Nelson rules 1–4 and (B) combined stability metrics is plotted as a function of the number of subgroups (x-axis) for datasets where a mean shift was introduced in the last 15 observations using an XmR chart. Results of simulations for mean shifts of increasing magnitude are overlaid.

Nelson Rules 1- 4

SR Test & ANOVA

Britt, Ramirez and Mistretta (2016)

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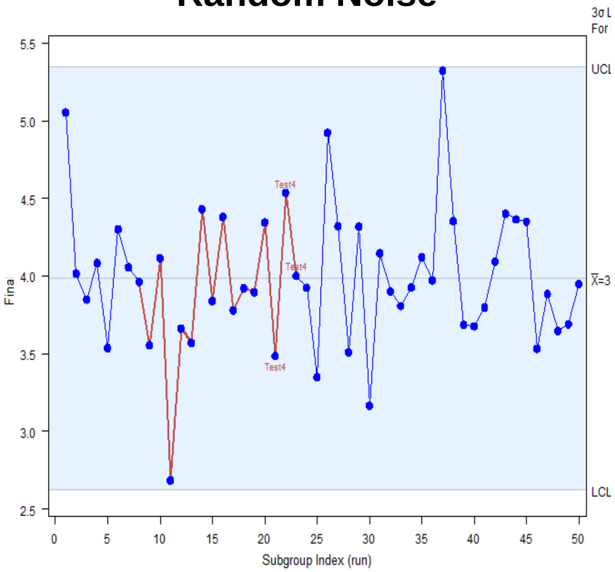
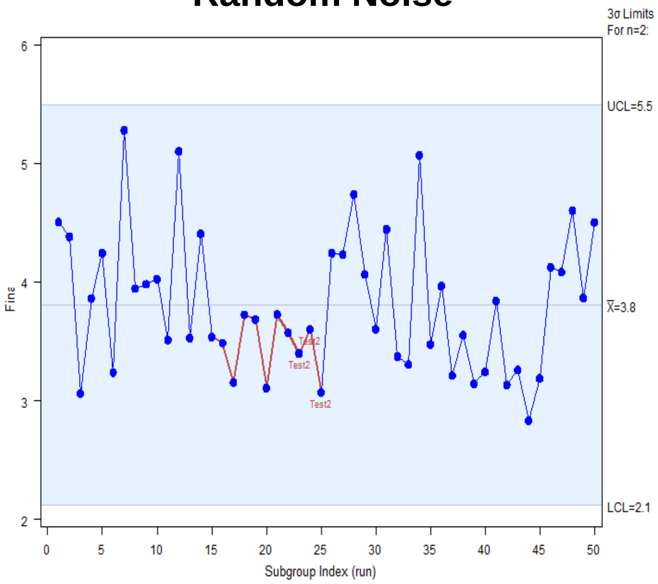
CHALLENGES USING SHEWHART CHARTS & NELSON RULES

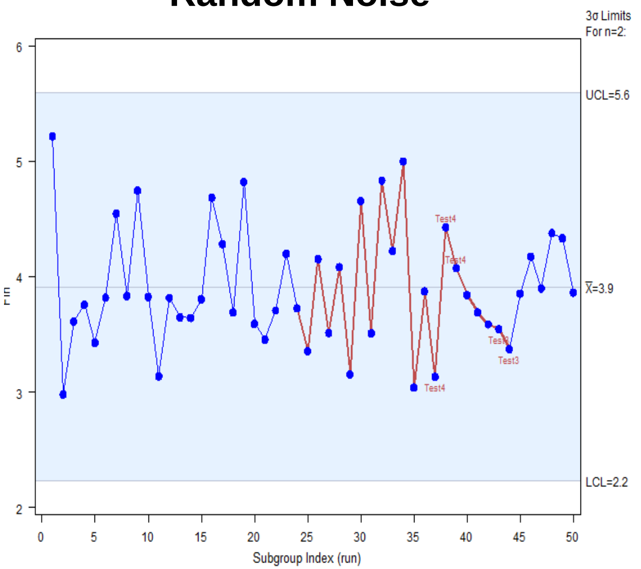
Random Noise



Random Noise



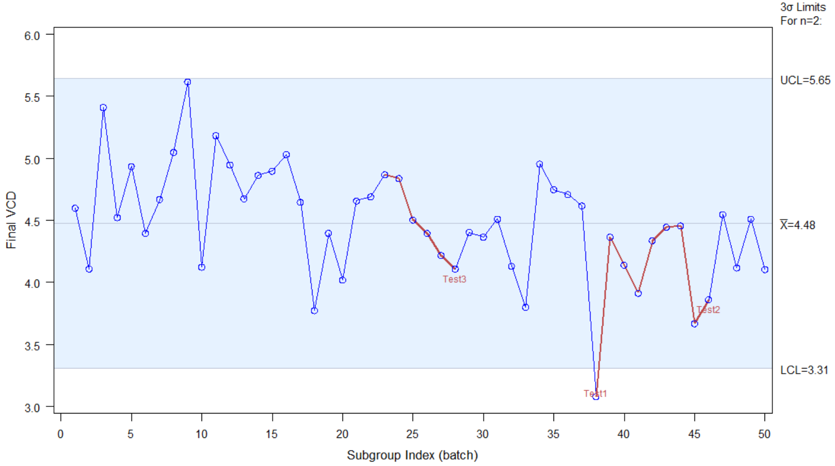
Random Noise



What is causing these patterns: ***common*** or ***special cause*** variation?

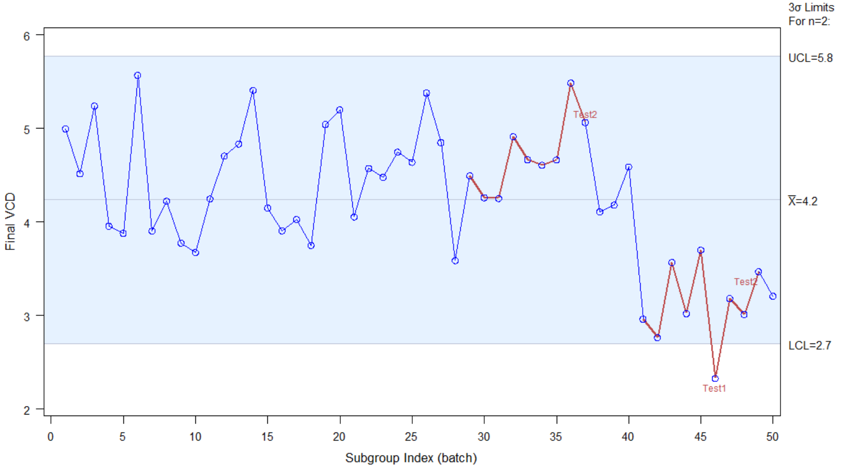
STABILITY METRICS TO INFORMING NELSON VIOLATIONS

Random Noise
Common Cause Variation



SR = 1.40, p-value = 0.1629
F = 1.32, p-value = 0.2475

Deliberate Mean Shift
Special Cause Variation



SR = 2.19, p-value = 0.0118
F = 3.63, p-value = 0.0011

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COMBINING PROCESS STABILITY & PROCESS CAPABILITY

	Stable YES	Stable NO
Capable YES	***Ideal State*** Process in control and 100% conformance.	***Predictability Issue*** All may seem OK, but . . .
Capable NO	***Yield Issue*** Must change process or readjust specifications.	***Double Trouble*** First eliminate special causes from process. Then re-engineer.

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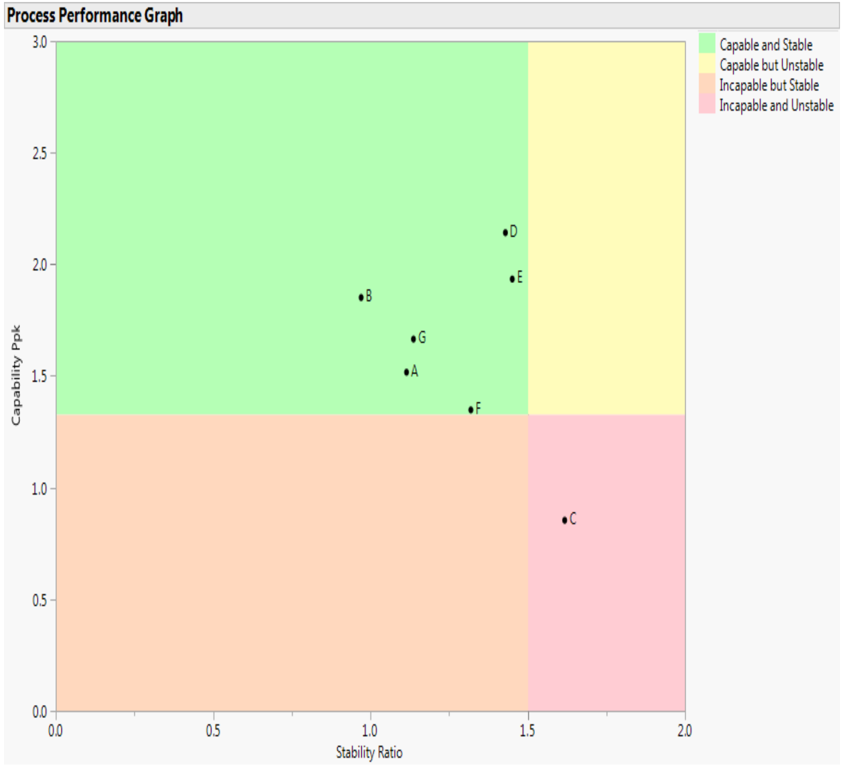

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SUMMARY

- Three measures and test criteria were developed for classifying process output as ‘stable’ or ‘unstable’
  - SR Test
  - ANOVA F-test
  - I_{NSR}
- Quantitative measures of process stability can be used to inform Nelson violations during Phase I monitoring
- Stability and capability assessments can be combined to provide an overall assessment of Process Health using the Process Performance Graph in JMP.

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ACKNOWLEDGEMENTS

- George Runger
- Donald Wheeler
- José Ramírez
- Keith Britt
- Tom Mistretta
- Mark DiMartino
- Dan Weese

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