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Stability Assessment with the Stability Index

JOHN SZARKA

WILLIS JENSEN

KEVIN WHITE

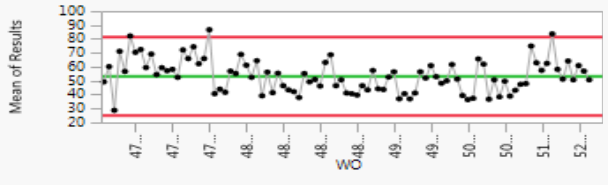
Scenario

Quality manager wants to assess the capability and stability for all the products that are produced in a manufacturing facility

Where should process improvement efforts be undertaken?

Presummarize Control Chart Item=Item 07

Individual on Group Means of Results



Presummarize Control Chart Item=Item 19

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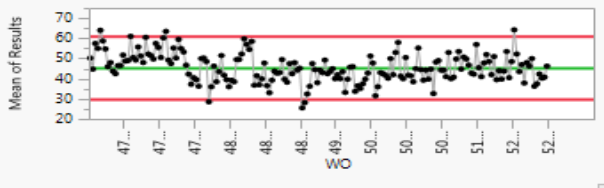

Presummarize Control Chart Item=Item 47

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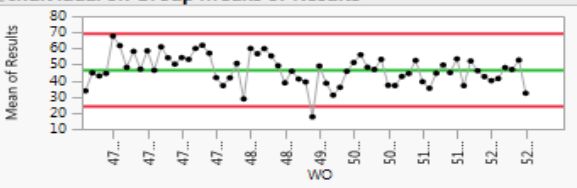

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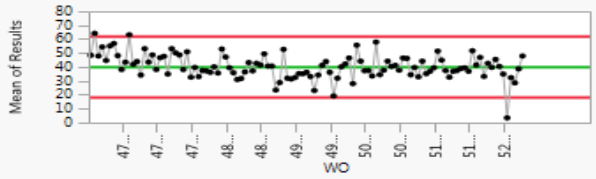
Individual on Group Means of Results



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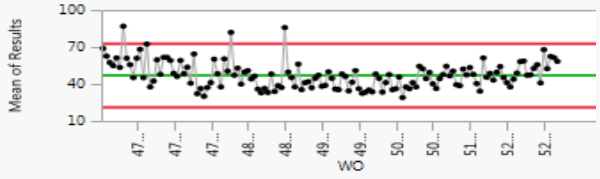

Presummarize Control Chart Item=Item 69

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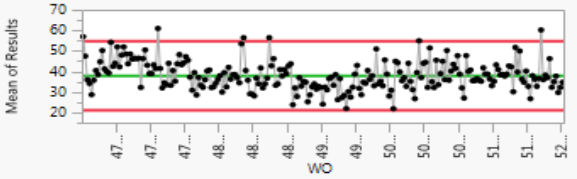
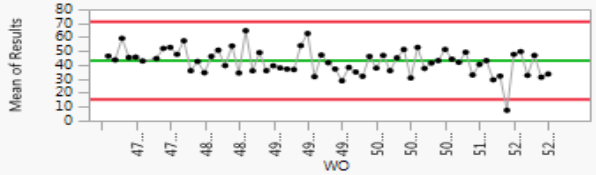


Presummarize Control Chart Item=Item 11

Individual on Group Means of Results



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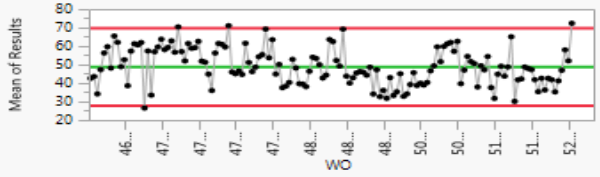
Individual on Group Means of Results



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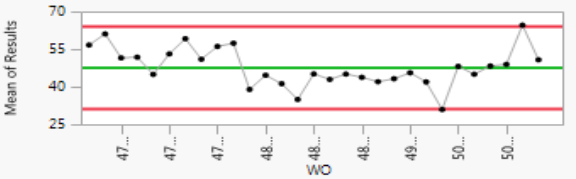
Individual on Group Means of Results



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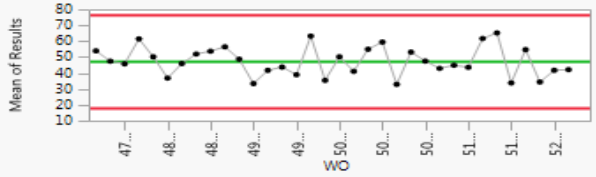
Individual on Group Means of Results



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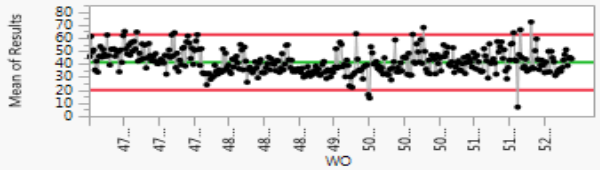
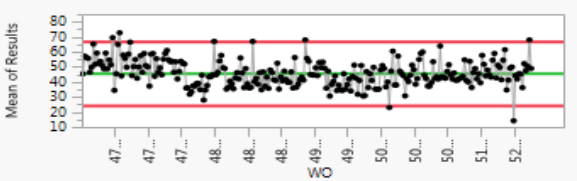


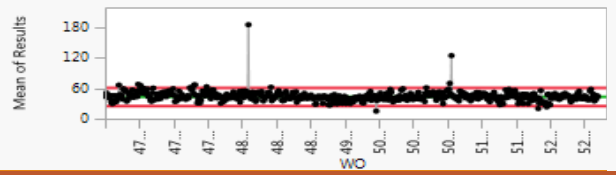
Presummarize Control Chart Item=Item 78

Individual on Group Means of Results



Presummarize Control Chart Item=Item 15

Individual on Group Means of Results



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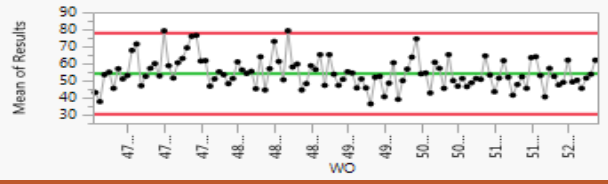

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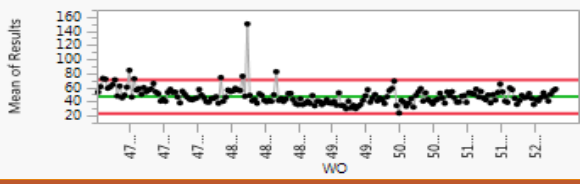


Problem

How does one quickly sift through large amounts of information to determine area of focus?

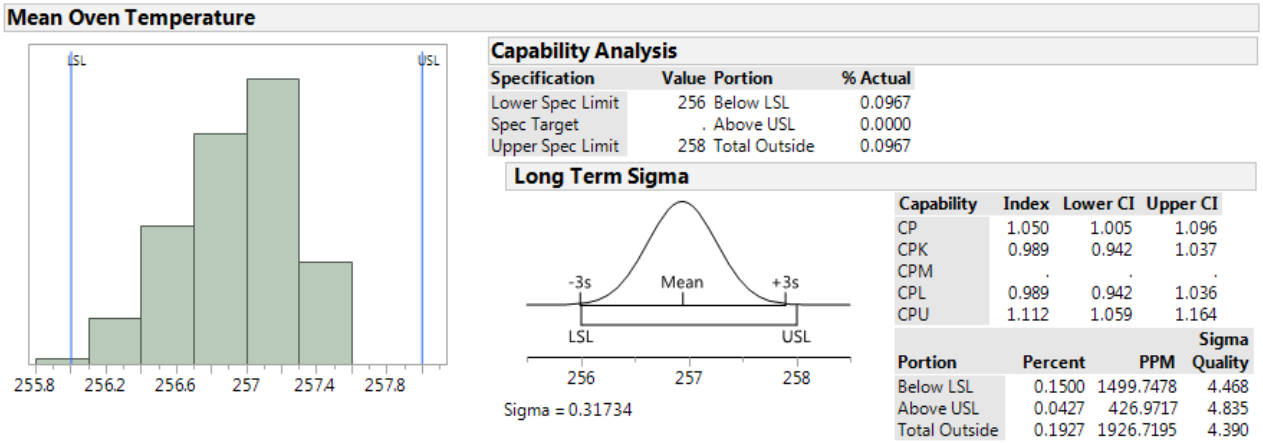
Described nicely by Stephen Few in *Signal: Understanding What Matters in a World of Noise*

More specifically in the quality arena, how do you best summarize process stability in a single metric?

Stability and Capability Indices

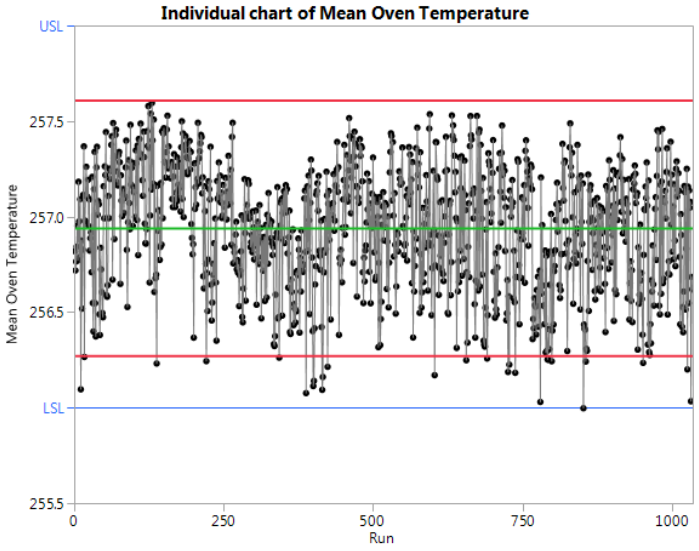
Capability

Supplement the histogram with a capability index (eg Cpk)



Stability

Supplement the control chart with ??



??

Previous Work on Summary Metrics

Cruthis and Rigdon (1992) and Ramirez and Runger (2006)
- Stability Ratio (SR) = Long-term variance/ Short-term variance

Ramirez and Runger (2006)
- Instability Ratio (INSR) = Percentage of control chart groups that violate Western Electric runs rules

Ramirez and Runger (2006)
- ANOVA = Hypothesis test based on artificial grouping of points to compare long-term and short-term variation

Gauri (2010)
- Process Stability Indicator (PSI) = complex calculation involving sums of squares of errors for least square regression lines

Sall (2017)
- Utilizes SR in JMP 13 Process Screening platform. Additional sensitivity indicators and parameter estimation methods discussed.

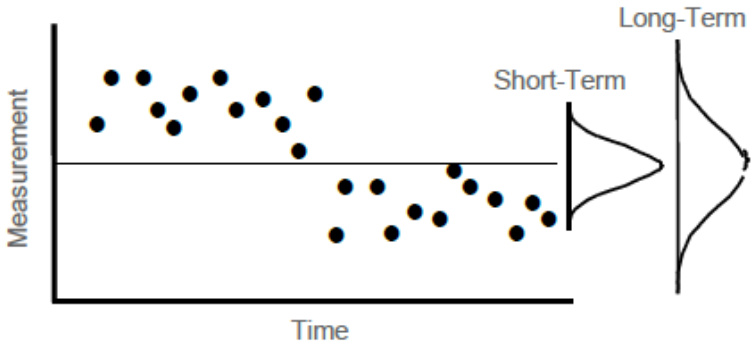
We want a metric that meets three key criteria (simple to calculate, easy to interpret, direct connection to capability indices)

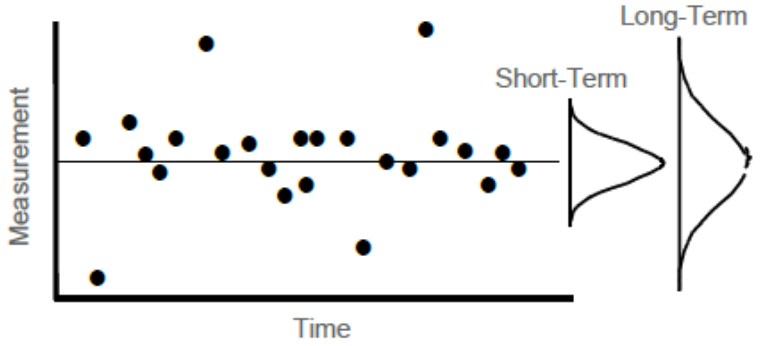
Our Approach = The Stability Index

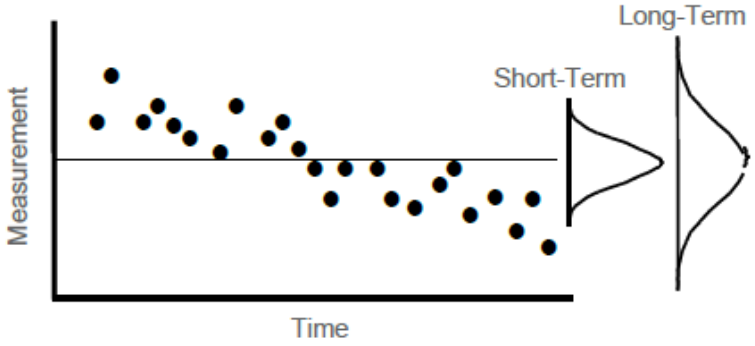
Modify the Stability Ratio to compute the ratio of the long-term **standard deviation** to the short-term **standard deviation**

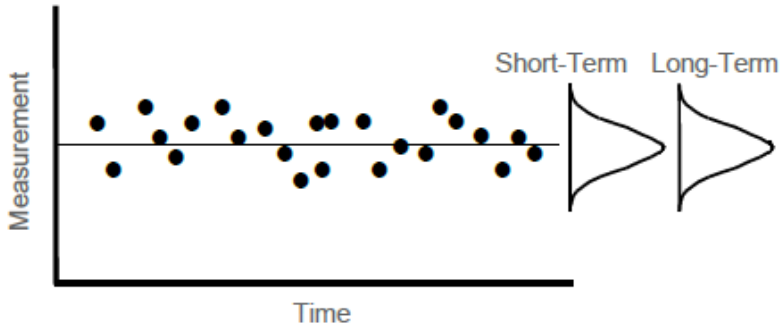
$$SI = \frac{\sigma_{LT}}{\sigma_{ST}}$$

Comparing LT & ST Estimates

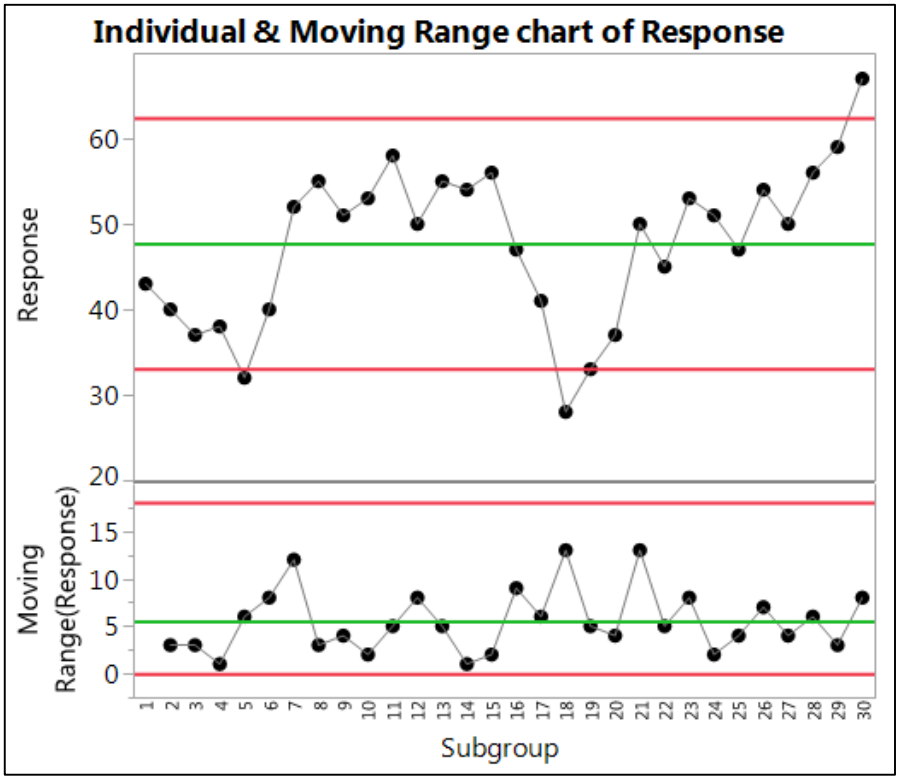




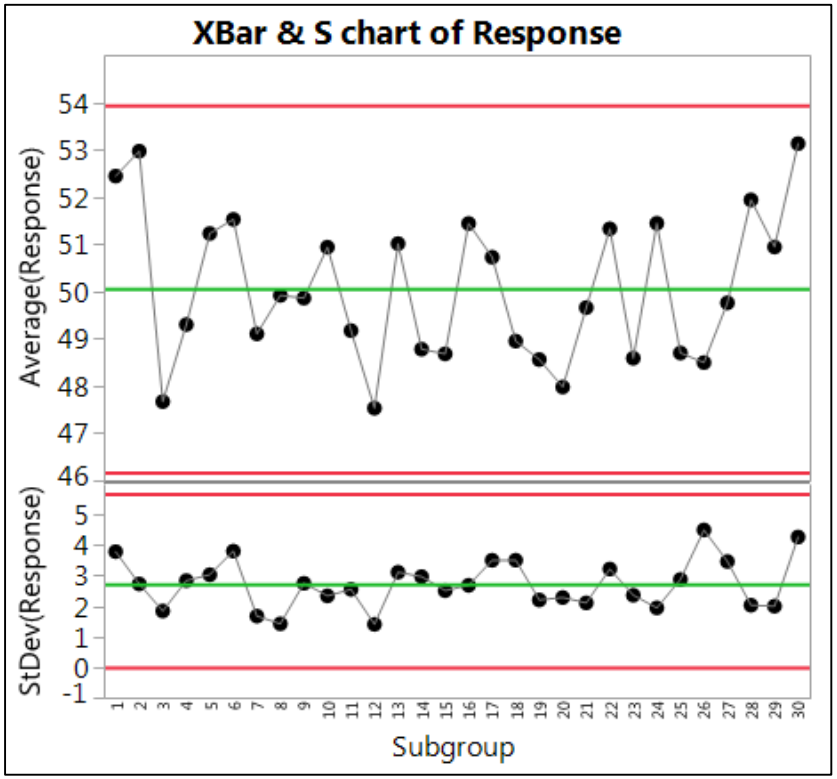




I/MR Example – SI = 1.87



Xbar/S Example – SI = 1.03



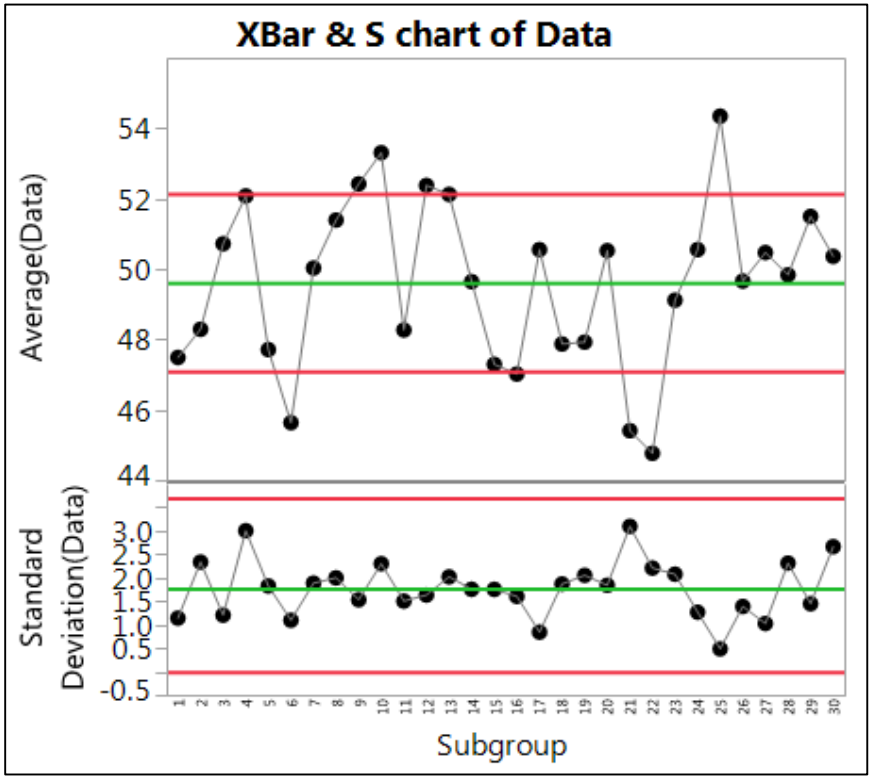
3-way Example

Some processes have expected between-subgroup variation (want to treat it as common cause)

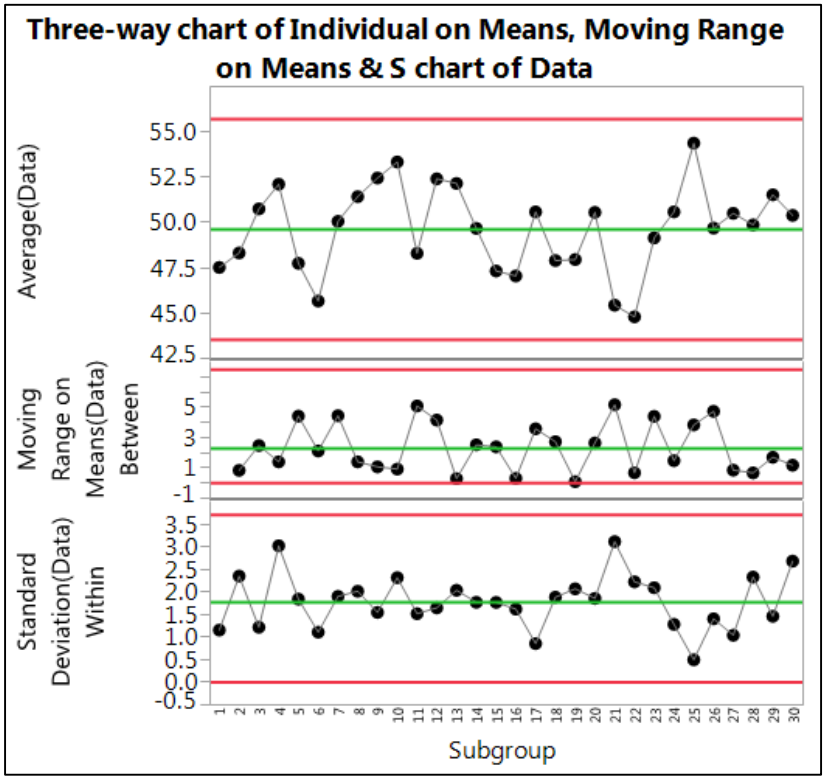
Batch processing a popular example

Xbar limits are too tight

Need to use a 3-way chart in this case rather than the Xbar/S chart



3-way Example – SI = 1.09



ST Standard Deviation Estimates

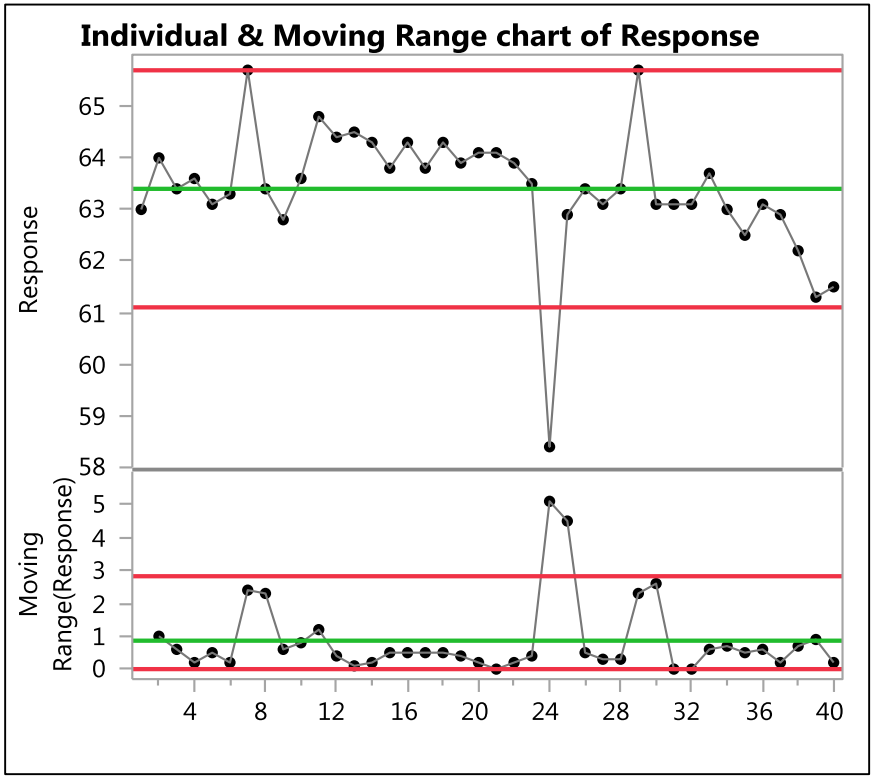
Subgroup Size	Expected Variation Between Subgroups?	Control Chart	Short-term Standard Deviation ($\hat{\sigma}_{ST}$)
1	--	IR	$\overline{MR}/d_2$
>1	No	$\bar{X}$ & s	$\bar{s}/c_4$
>1	No	$\bar{X}$ & R	$\bar{R}/d_2$
>1	Yes	Three-Way (I on means, MR on means, s on within)	$\sqrt{\left(\frac{\overline{MR}}{d_2}\right)^2 + \left(\frac{\bar{s}}{c_4}\right)^2 \left(1 - \frac{1}{n}\right)}$
>1	Yes	Three-Way (I on means, MR on means, R on within)	$\sqrt{\left(\frac{\overline{MR}}{d_2}\right)^2 + \left(\frac{\bar{R}}{d_2}\right)^2 \left(1 - \frac{1}{n}\right)}$

Robust Estimation

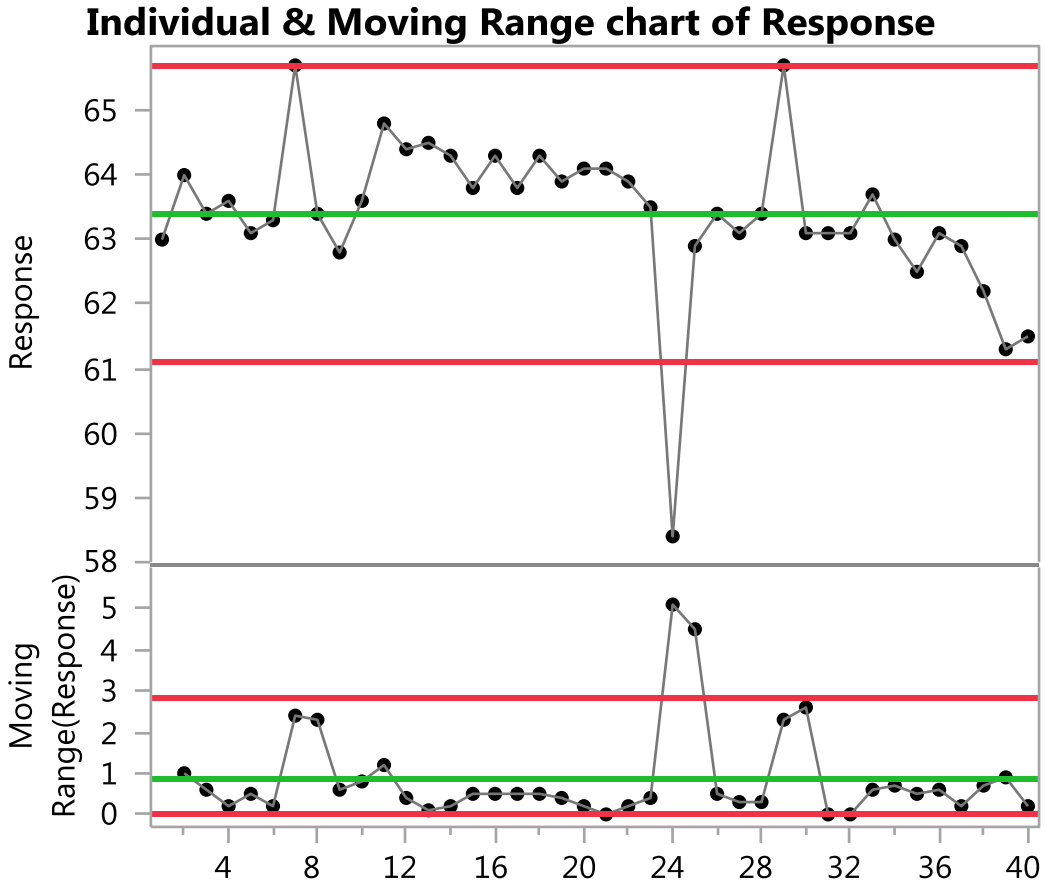
Most prevalent with individual charts

May elect to exclude data where MR values exceed thresholds

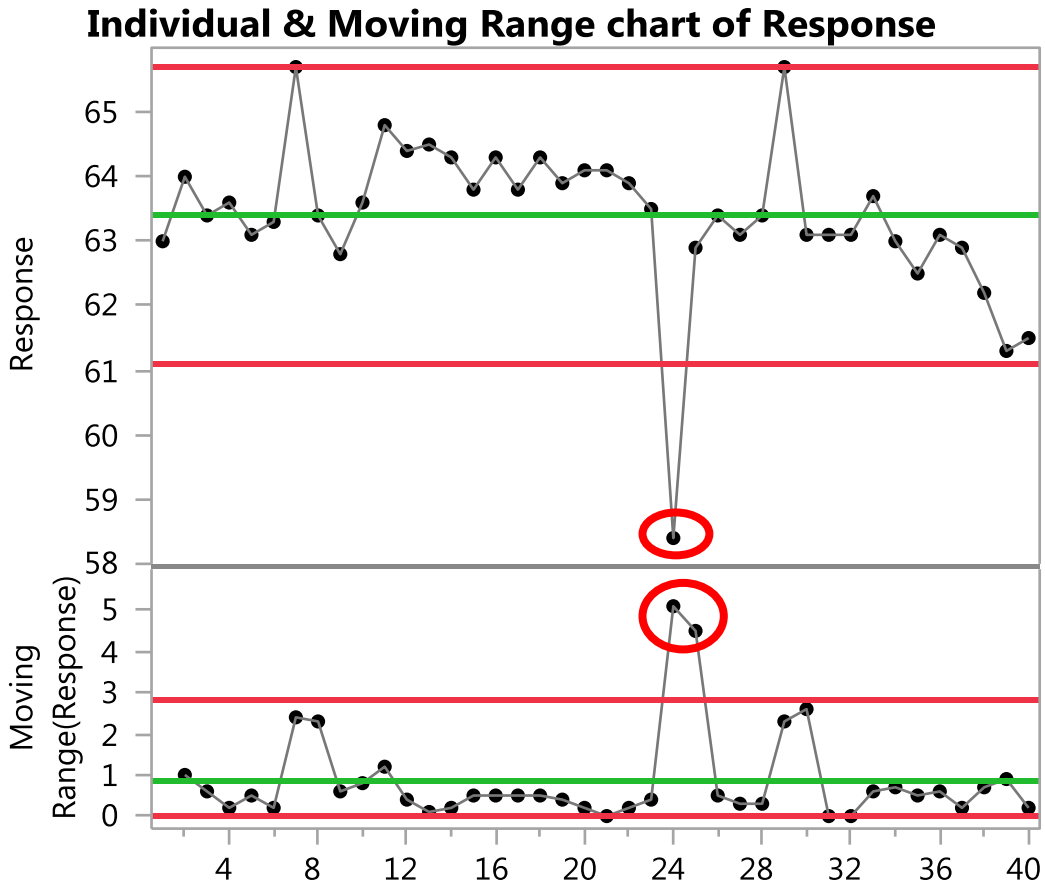
Median MR instead of Average MR



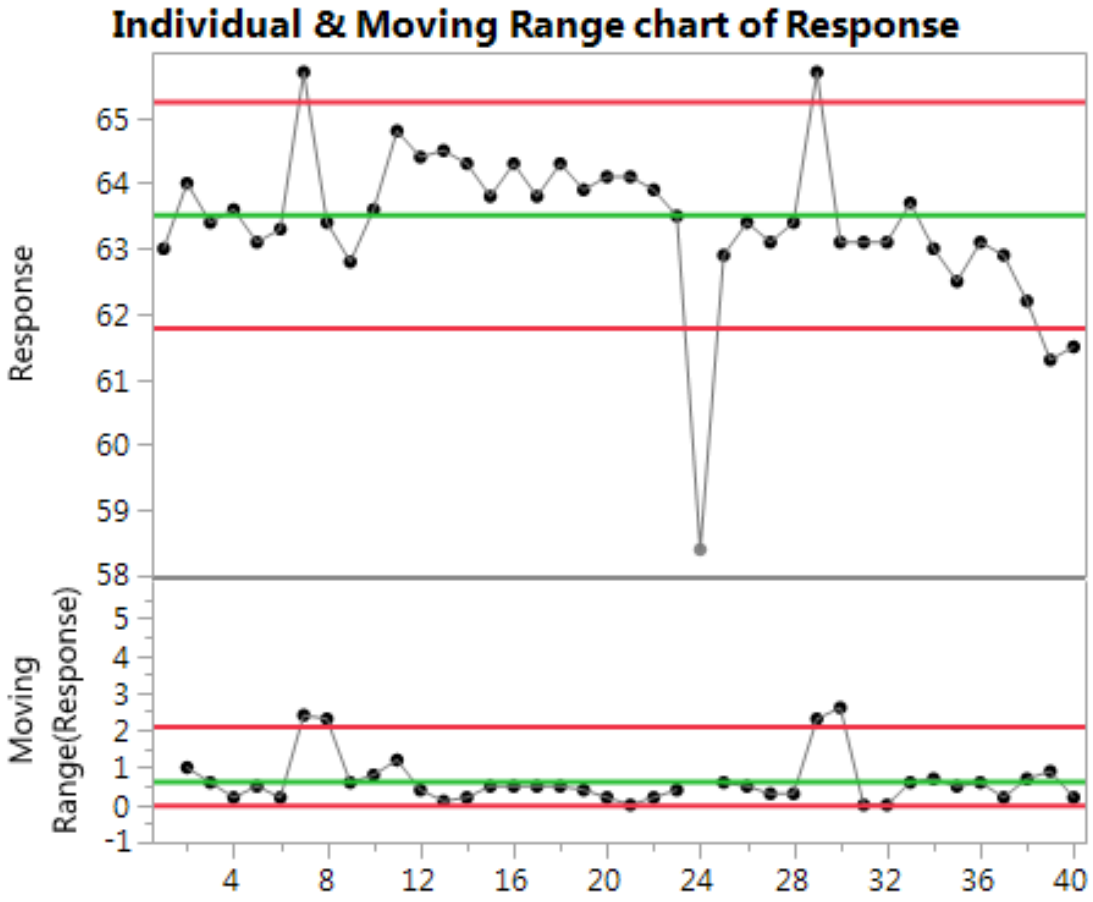
Original – SI = 1.57



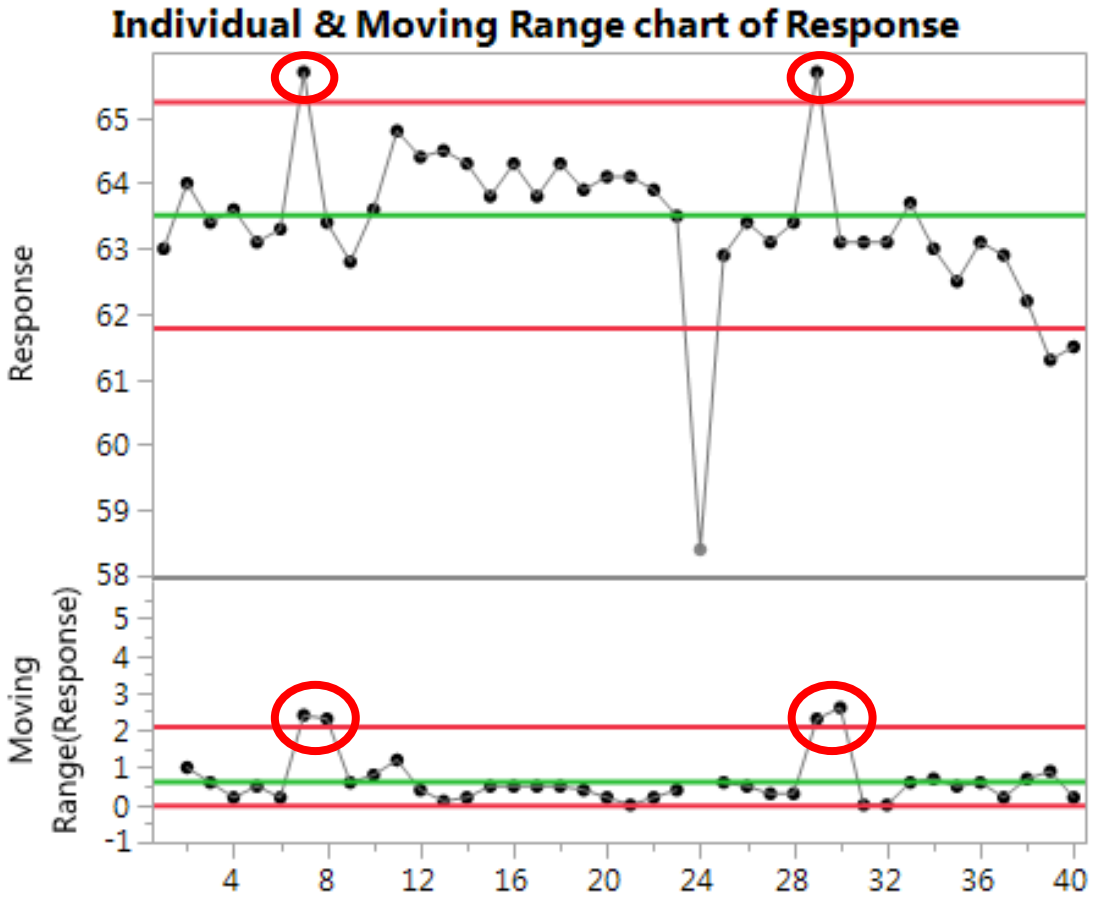
Original – SI = 1.57



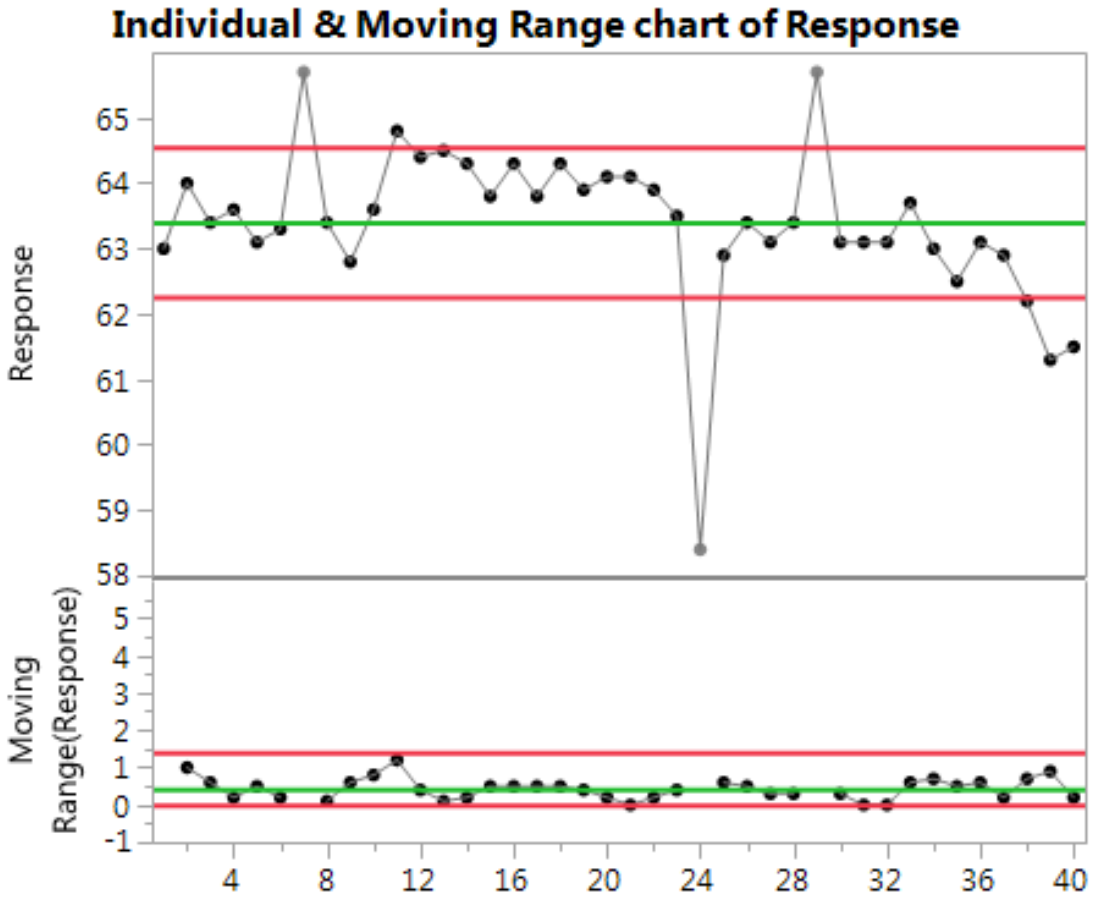
Without Largest Outlier – SI = 2.09



Without Largest Outlier – SI = 2.09



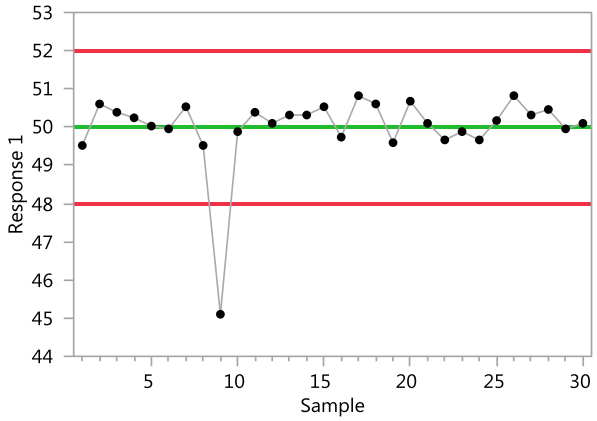
Without Additional Outliers – SI = 3.09

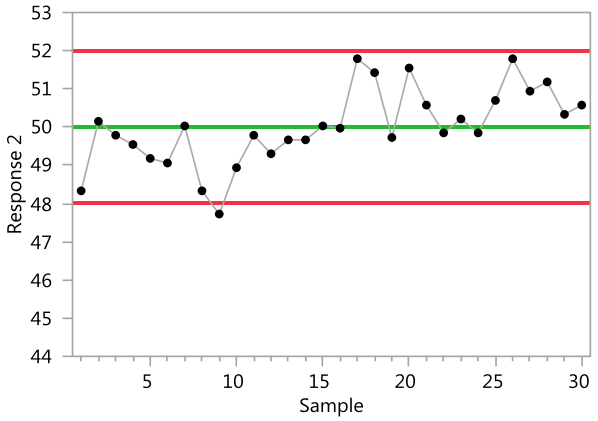


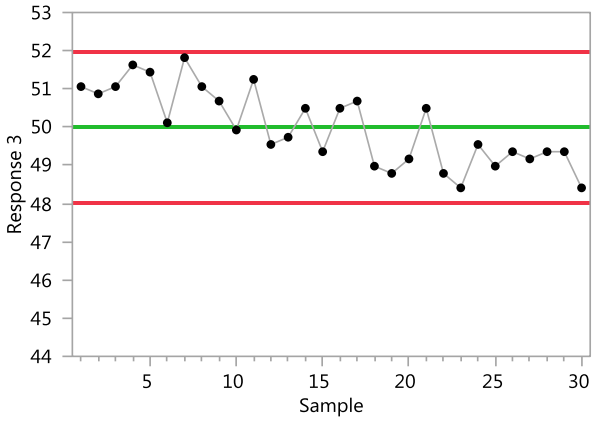
Index ≠ Chart

All 3 processes have SI = 1.5 despite exhibiting very different data streams

The risk of misclassifying a process based only on the SI is similar to what is done for other indices, such as Ppk





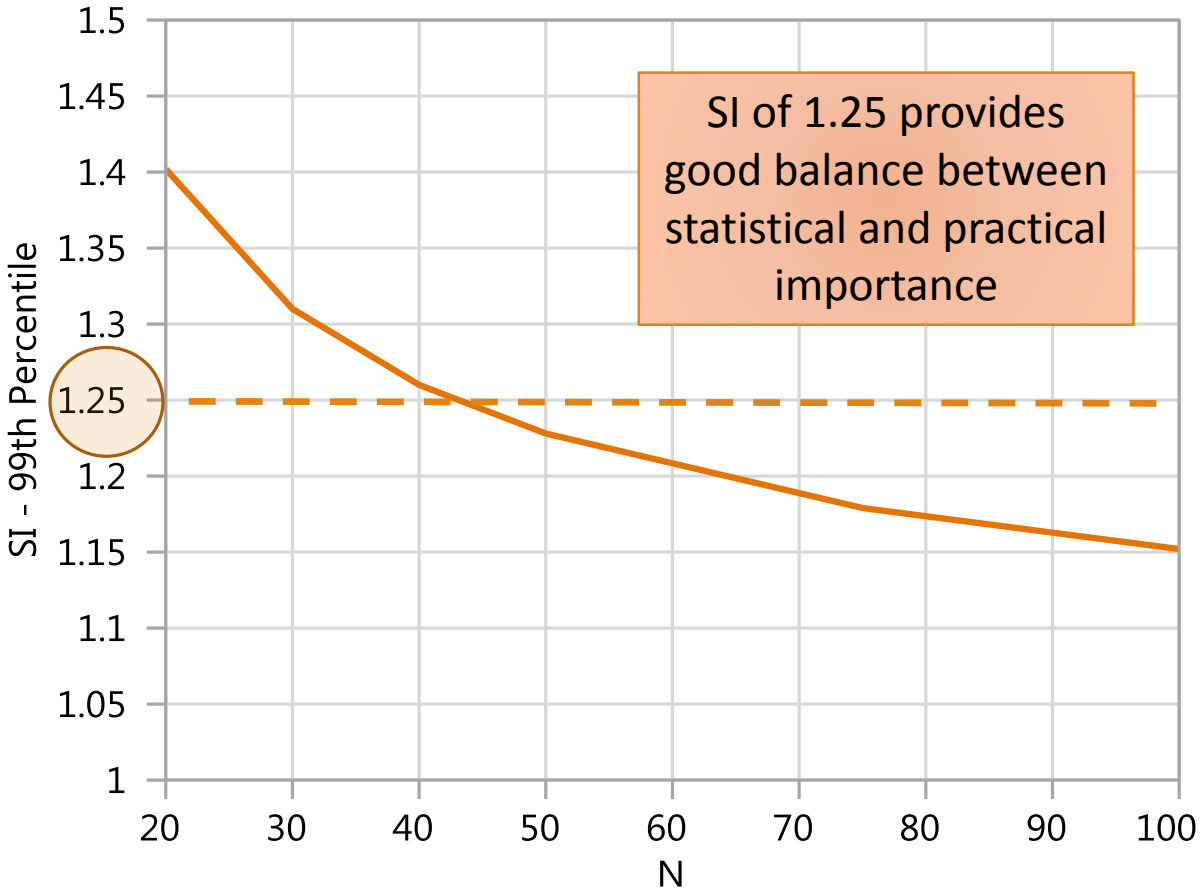


SI Connection to Capability

The stability index (SI) can be conveniently expressed as a function of common process capability and performance indices

$$SI = \frac{\sigma_{LT}}{\sigma_{ST}} = \frac{\boldsymbol{C_{pk}}}{\boldsymbol{P_{pk}}} = \frac{\boldsymbol{C_p}}{\boldsymbol{P_p}}$$

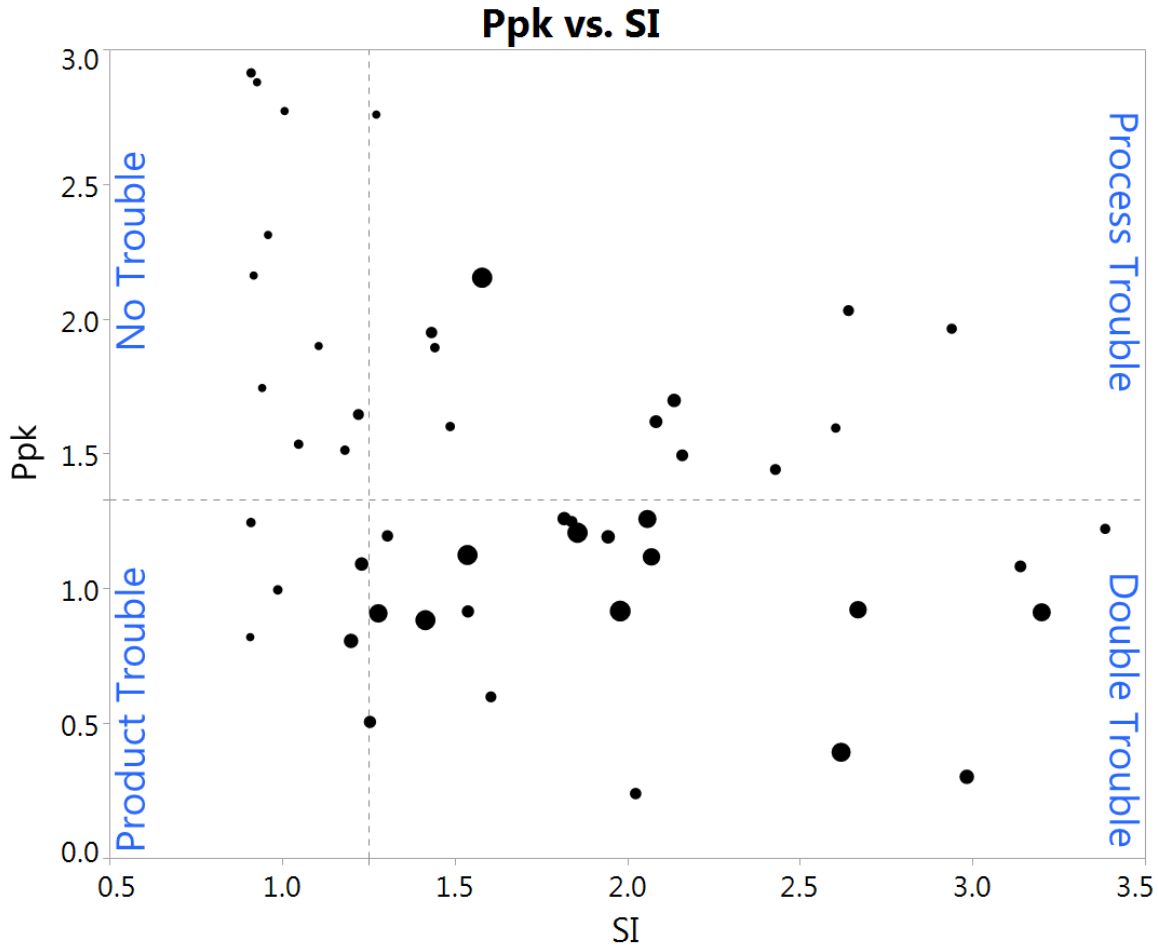
SI Rule of Thumb



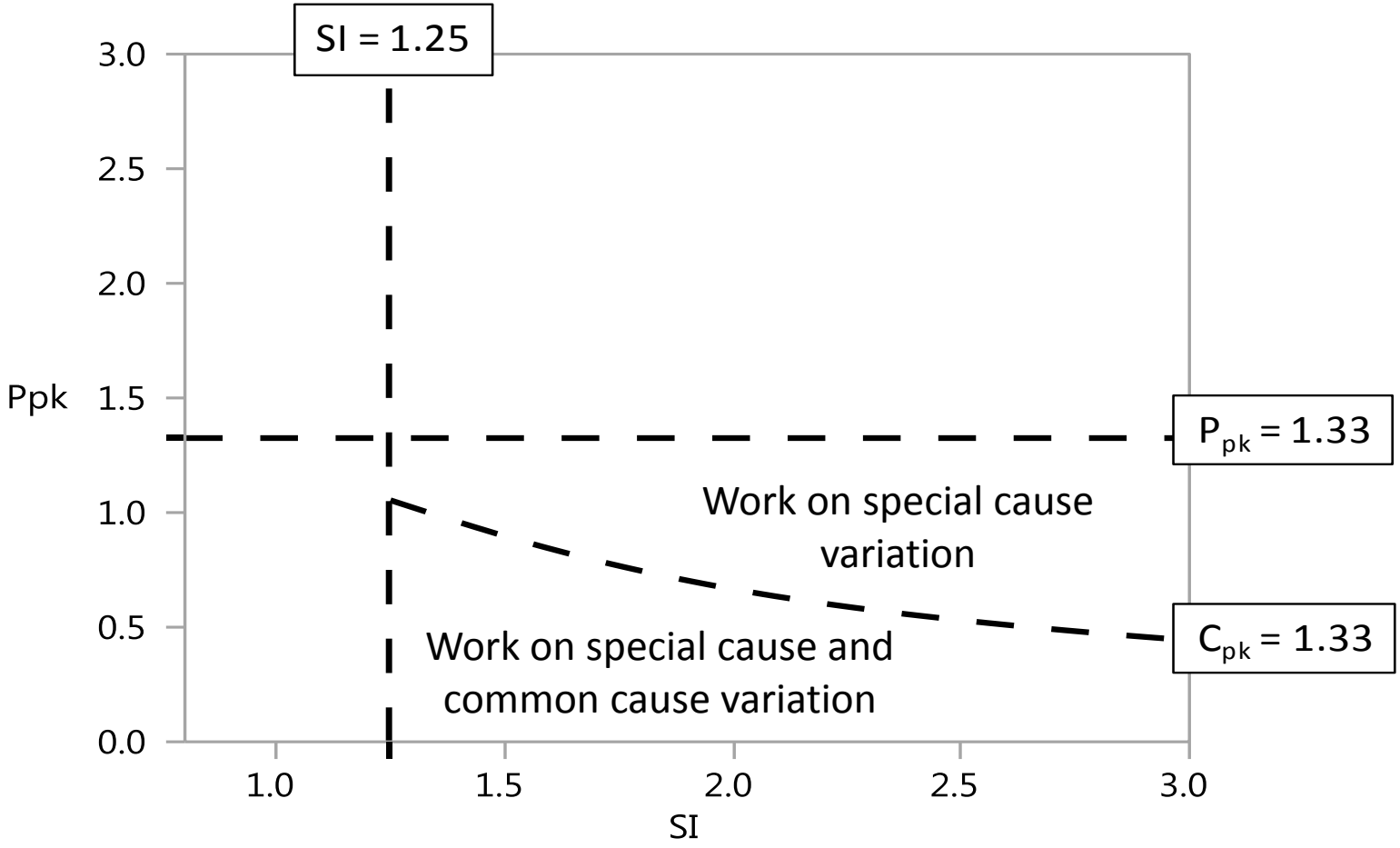
Four Process States (Wheeler)

Process State	SI and Capability Rules of Thumb
No Trouble (Ideal State)	SI < 1.25 P_{pk} > 1.33
Process Trouble (Brink of Chaos)	SI > 1.25 P_{pk} > 1.33
Product Trouble (Threshold State)	SI < 1.25 P_{pk} < 1.33
Double Trouble (State of Chaos)	SI > 1.25 P_{pk} < 1.33

Process Performance Graph



Enhanced Process Performance Graph



Stability Index Advantages

The SI is simple to calculate, easy to interpret, and directly connected to capability

Stability Assessment Approach	Simple to Calculate	Easy to Interpret	Connected to Capability
INSR	No	Yes	No
PSI	No	No	No
ANOVA	No	No	No
SR	Yes	Partial	Partial
SI	Yes	Yes	Yes

Conclusions/Take Home Message

The SI combined with combined with capability indices (Ppk) can quickly help assess numerous processes and identify which need improvement

Additionally, the type of improvement needed is identified

- Stability (special cause), Capability (common cause), or both

Visualization is easy using the process performance graph

Future Work/Other Potential Indices

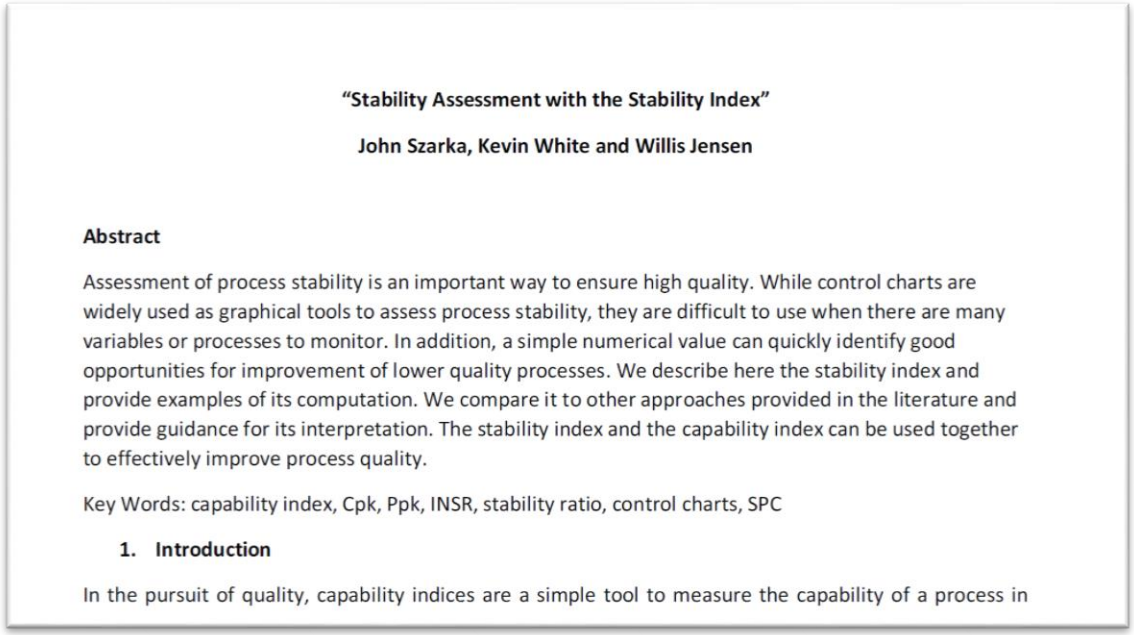
Target Index (TI) – The number of short-term standard deviations the process average is from target

$$TI = 3(C_p - C_{pk})$$

Measurement System Indices – White and Borror (2011) addressed numerous measurement system and their connection to capability, and recommend guidelines for when to improve the actual process or measurement system

For More Information

1. Paper submitted to *Quality Engineering*



"Stability Assessment with the Stability Index"

John Szarka, Kevin White and Willis Jensen

Abstract

Assessment of process stability is an important way to ensure high quality. While control charts are widely used as graphical tools to assess process stability, they are difficult to use when there are many variables or processes to monitor. In addition, a simple numerical value can quickly identify good opportunities for improvement of lower quality processes. We describe here the stability index and provide examples of its computation. We compare it to other approaches provided in the literature and provide guidance for its interpretation. The stability index and the capability index can be used together to effectively improve process quality.

Key Words: capability index, Cpk, Ppk, INSR, stability ratio, control charts, SPC

1. Introduction

In the pursuit of quality, capability indices are a simple tool to measure the capability of a process in

2. Next session in the FTC program (Ramirez, 5C)

References

Britt, K.A., Ramírez, B. and Mistretta, T. (2016). Process monitoring using statistical stability metrics: Applications to Biopharmaceutical Processes. *Quality Engineering*, 28(2), pp. 193-211.

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